Dislocated Actuator/Sensor Positioning and Feedback Design for Flexible Structures

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A method is presented which allows an integrated determination of actuator/sensor positions and feedback gains for control of flexible structures. This method is based on the maximization of dissipation energy due to control action. The optimality criterion is determined via a Liapunov equation, and it is maximized with a recursive quadratic programming algorithm. The application of this method to a simple flexible structure yields several in-general dislocated actuator and sensor locations, which are locally optimal. An extension of the method to treat spillover effects is implemented as an additional constraint to the optimization criterion.

Nomenclature

$A', \tilde{A}$	= dynamic matrices
$B(x_a), B^*(x_a), B'$	= input matrices
c_i	= optimization constraints
$C(x_s), C^*(x_s), C'$	= measurement matrices
d	= vector of search direction
D_0, D_c, D_c^*	= damping matrices
f	= vector of input forces
F	= optimization criterion
G_c	= gyroscopic matrix
H	= approximation of the Hessian
J_e, J_i	= set of equality and inequality constraints
k_{ij}	= elements of feedback matrices
K, K^*	= feedback matrices
K_S	= stiffness matrix
L	= beam length
$\mathfrak{L}$	= Lagrangian
m	= number of input forces
M	= mass matrix
n, n^*	= number of modes
P	= solution matrices of a Liapunov equation
q	= vector of generalized coordinates
Q	= weighting matrix
r	= number of velocity sensors
R	= test function
t	= time
v	= vector of velocity measurements
$w(x,t)$	= elastic displacement
W_i	= energy
$x, (x)$	= (vector of) space coordinates
$x_{a_i}, (x_a)$	= (vector of) actuator coordinates
$x_{s_i}, (x_s)$	= (vector of) sensor coordinates
y	= optimization vector
α_k	= relaxation factor
η_i	= efficiency
$(\phi), \phi_i$	= (vector of) admissible functions
λ	= Lagrange multiplier
∇	= gradient operator
ρ	= convergence parameter
ξ_i	= damping factor
ω_i	= eigenfrequencies

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I. Introduction

POSITIONING of sensors and actuators for the control of technical systems is guided by physical considerations. Starting from the structure of the plant, one tries to measure directly by appropriate sensors the variables to be controlled and to install actuators in such a way that they are most efficient and only small forces and torques act on the plant. In this way, efficient superpositioned control loops result, where large masses are moved slowly (coarse control, low frequent) and small masses are moved fast (fine control, high frequent).

The same considerations are valid for the control of space vehicles consisting of rigid bodies with coupled elastic substructures. However, for vibration isolation of an elastic structure, this methodology is not sufficient. Here it is not the relative motion of different bodies that is controlled but the relative motion of an infinite number of mass elements of a single body. The motion of the elastic structure as a continuum is described by partial-differential equations, which depend on internal properties like mass distribution, damping, stiffness, and external boundary conditions.

The final goal of the control of elastic modes is to increase damping and/or stiffness of the structure so as to achieve the desired time behavior. This behavior depends on the number and location of sensors and actuators as well as on the chosen structure of the controller. Up to now, the problem of positioning the actuators and sensors has been solved mainly via minimization of the control energy using controllability measures, respectively integral criteria.[1-5] Thereby only a separate determination of actuator locations and sensor locations is possible. Furthermore, the influence of the feedback gain cannot be considered.

In this paper a new method is presented that allows an integral determination of actuator/sensor positions and feedback gains via maximization of the dissipation energy, which is extracted by action of the feedback system.

The idea of this procedure is presented in Sec. II and is applied to a simple structure in Sec. III. Then in Sec. IV, extensions of the proposed method are given, which allow an efficient reduction of spillover effects.

II. Dissipation Energy Maximization

A. System Modeling

The motion of continuous flexible structures can be described by a set of coordinates depending on both space and time, thus leading to partial-differential equations in the elastic displacement $w(x,t)$. Here the coordinate $w(x,t)$ represents the space- and time-dependent deformation of the structure, which, in general, may also contain deformations in the y and z direction. This does not restrict the applicability of the method presented. The solution of the homogeneous part of the underlying partial-differential equation is done using the separation principle

$$w(x,t) = \sum_{i=1}^{n} \phi_i(x) q_i(t) = \phi^T(x) q(t) \tag{1}$$

where $\phi(x)$ is a vector of space-dependent admissible functions solving the eigenvalue problem (mode shapes) and $q(t)$ is the vector of time-dependent generalized coordinates. The number n of the relevant modes is truncated at an "appropriate" figure.

Then the time-differential equation of the generalized coordinate vector q

$$M\ddot{q} + D_0\dot{q} + K_s q = B(x_a) f \tag{2}$$

describes the small motion of the structure about the equilibrium point. In Eq. (2) the $n \times n$ matrices M, D_0, and K_s are normalized diagonal matrices, where $M = \mathrm{diag}[1]$ represents the mass matrix, $D_0 = \mathrm{diag}[2\xi_i\omega_i]$ the damping matrix and $K_s = \mathrm{diag}[\omega_i^2]$ the stiffness matrix. For the m-dimensional force input vector f, the input matrix is determined as

$$B(x_a) = [\phi(x_{a_1}) \cdots \phi(x_{a_m})] \tag{3}$$

with x_{a_j} the actual position of the corresponding force actuator. The r-dimensional velocity measurement v is then determined by the equation

$$v = C(x_s)\dot{q} \tag{4}$$

with

$$C(x_s) = \begin{bmatrix} --\phi^T(x_{s_1})-- \\ \cdot \\ \cdot \\ \cdot \\ --\phi^T(x_{s_r})-- \end{bmatrix} \tag{5}$$

and x_{s_p} the actual position of the corresponding sensor device.

If a finite-element approach is applied for modeling of the flexible structure, analytic expressions for the mode shapes as functions of x_{a_j} and x_{s_p} have to be determined. This can be done, for example, by using spline approximations.

As controller for this system, constant output vector feedback

$$f = -K \cdot v \tag{6}$$

is considered, with K as an $m \times r$ time invariant matrix

$$K = \begin{bmatrix} k_{11} \cdots k_{1r} \\ k_{m1} \cdots k_{mr} \end{bmatrix} \tag{7}$$

The goal of the control system design proposed in this paper is to determine actuator locations x_{a_j}, sensor locations x_{s_p}, and feedback gains k_{jp} in one integrated design procedure. The crucial point of the design problem is the choice of the appropriate criterion, which accumulates the effects of actuator positions, sensor positions, and feedback gains as well.

B. Formulation of the Energy Criterion

To avoid the arbitrariness of weighting of the state and control terms in standard Riccati design, a physically significant optimization criterion was used for actuator/sensor positioning and feedback determination. The criterion chosen is an energy criterion which is determinable using a time-saving Liapunov solution, as shown in the following equations.

The total energy stored in the system is the sum of kinetic and potential energy

$$W_F = T + V = \tfrac{1}{2}\dot{q}^T M \dot{q} + \tfrac{1}{2} q^T K_s q \tag{8}$$

Differentiation with respect to time and using Eq. (2) with $f = \mathbf{0}$

$$\dot{W}_F = \tfrac{1}{2}\ddot{q}^T M \dot{q} + \tfrac{1}{2}\dot{q}^T M \ddot{q} + \tfrac{1}{2}\dot{q}^T K_s q + \tfrac{1}{2} q^T K_s \dot{q} = -\dot{q}^T D_0 \dot{q} \tag{9}$$

yields the energy rate extracted from the system by internal damping agencies. Reintegration leads to

$$W_F = -\int_{t_0}^{\infty} \dot{q}^T D_0 \dot{q} \, \mathrm{d}t \tag{10}$$

which is called dissipation energy of the system. As D_0 is the matrix of modal damping, W_F represents the energy dissipated by internal velocity proportional friction. Because of the absence of other dissipation factors, W_F is equal to the total initial energy W_0.

Considering constant velocity feedback,

$$f = -Kv = -KC(x_s)\dot{q} \tag{11}$$

now adds a new velocity proportional term to Eq. (2)

$$M\ddot{q} + [D_0 + B(x_a)KC(x_s)]\dot{q} + K_s q = 0 \tag{12}$$

This additional term splits into a symmetric dissipative part

$$D_c = \tfrac{1}{2}\{B(x_a)KC(x_s) + [B(x_a)KC(x_s)]^T\} \tag{13}$$

and a skew symmetric matrix of conservative forces

$$G_c = \tfrac{1}{2}\{B(x_a)KC(x_s) - [B(x_a)KC(x_s)]^T\} \tag{14}$$

with $D_c + G_c = BKC$. The matrix D_c represents the damping induced by the velocity feedback and G_c describes the gyroscopic effects of the feedback.

Analog to Eq. (10), the integral

$$W_c = -\int_{t_0}^{\infty} \dot{q}^T D_c \dot{q} \, \mathrm{d}t \tag{15}$$

represents the energy which is dissipated by the control action. The initial energy W_0 of the system is now partially dissipated by friction and control

$$W_0 = W_F + W_c \tag{16}$$

The gyroscopic matrix G_c thereby takes care of the "internal" energy transfer between different modes of the system, without changing the total energy of the system.

Assuming that the internal damping ξ_i of flexible structures is rather small (up to approximately 2-5%), thus leading to long decay times of the structural oscillations, the idea is to increase the energy dissipation through the action of control W_c. Implicitly it is assumed that an increase of W_c reduces oscillations and decay times of all modes, because no explicit dependence exists.

The dissipation energy W_c depends on the locations of the actuators x_a, the locations of the sensors x_s, and the feedback matrix K, and therefore it can be applied as an optimization criterion to determine both positions and feedback gains,

$$\max_{x_a,x_s,K} W_c(x_a,x_s,K) \longrightarrow x_a^*,x_s^*,K^* \tag{17}$$

with constraints

$$x_a \epsilon X_a, x_s \epsilon X_s, (K \epsilon K) \tag{18}$$

where X_a and X_s are subspaces which restrict x_a and x_s to lie on the flexible structure, and optionally K may pose some upper bounds on the feedback K.

This optimization is only feasible if the system a priori possesses internal damping (i.e., $D_0 > 0$); otherwise the optimization terminates if a stable solution is found. For any stable solution then, $W_c \equiv W_0$ is valid.

C. Numerical Determination of Optimal Actuator/Sensor Locations and Feedback Gains

1. Energy Determination

Transforming Eq. (2) to state space form yields

$$\begin{bmatrix} q \\ \dot{q} \end{bmatrix}^{\cdot} = \begin{bmatrix} 0 & I \\ -M^{-1}K_s & -M^{-1}D_0 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ B(x_a) \end{bmatrix} f$$

$$v = [0 \ C(x_s)] \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

or, in short form,

$$\dot{z} = A'z + B'f \qquad v = C'z \tag{19}$$

For the proposed output vector feedback [see Eq. (6)], the closed-loop stability matrix is given as

$$\tilde{A} = A' + B'KC' = \left[\begin{array}{c|c} 0 & I \\ \hline -M^{-1}K_s & -M^{-1}(D_0 + BKC) \end{array}\right] \tag{20}$$

Using the definition,

$$\tilde{Q} = \begin{bmatrix} 0 & 0 \\ 0 & D_c \end{bmatrix}$$

the controller-induced dissipation energy [Eq. (15)] can be written as

$$W_c = -\int_{t_0}^{\infty} \dot{q}^T D_c \dot{q} \mathrm{d}t = -\int_{t_0}^{\infty} [q^T \dot{q}^T] \tilde{Q} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \mathrm{d}t \tag{21}$$

Applying standard state transformation techniques, Eq. (21) yields

$$W_c = -[q^T(0) \dot{q}^T(0)] \int_{t_0}^{\infty} e^{\tilde{A}^T t} \tilde{Q} e^{\tilde{A}t} \mathrm{d}t \begin{bmatrix} q(0) \\ \dot{q}(0) \end{bmatrix} \tag{22}$$

The solution of the integral expression defined as

$$P = -\int_{t_0}^{\infty} e^{\tilde{A}^T t} \tilde{Q} e^{\tilde{A}t} \mathrm{d}t$$

is equivalent to the solution of the following Liapunov equation

$$\tilde{A}^T P + P\tilde{A} = \tilde{Q} \tag{23}$$

Thus the controller-induced dissipation energy is determined as

$$W_c = -[q(0)^T \dot{q}(0)^T] P \begin{bmatrix} q(0) \\ \dot{q}(0) \end{bmatrix} \tag{24}$$

The determination of the dissipation energy W_c of a flexible structure with n modes requires the solution of Liapunov Eq. (23) for $2n \times 2n$ matrices. The solution of Eq. (23) exists and is unique as long as the eigenvalues of $\tilde{A}$ have negative real parts.[6] The numerical solution of the Liapunov equation is done efficiently using the algorithm of Bartels and Stewart, as symmetry of $\tilde{Q}$ is guaranteed.[7] During optimization, its path is restricted to matrices $\tilde{A}$ having eigenvalues with negative real parts.

2. Initial Conditions

As the dissipation energy depends on the initial conditions of the flexible structure, realistic conditions have to be chosen. These conditions were assumed to be those which occur after impact of an impulse surface load acting on the structure in equilibrium. The impulse surface loads result from short orbit maneuvers of the structure. These disturbances are transformed to corresponding initial conditions of the state vector. The consideration of any other initial conditions, for example, those resulting from impulsive point loads at a specific location, is also possible.[8]

3. Numerical Optimization

The optimization problem of Eqs. (17) and (18) represents a nonlinear optimization with constraints. The constraints are inequality constraints in x_a, x_s, and K. Han and Powell propose the following globally convergent extension of the damped Newton method for this problem[9-12]: Generate a sequence $\{y^k\}$ converging to a local solution y^* by means of recursively solving the following quadratic programming problem

$$\min_d \{\nabla F(y)^T d + \tfrac{1}{2} d^T H d\}$$

$$\text{s.t.} \begin{cases} \nabla c_i(y)^T d + c_i(y) = 0 & \forall i \epsilon J_e \\ \nabla c_i(y)^T d + c_i(y) \geq 0 & \forall i \epsilon J_i \end{cases}$$

which yields the search direction d in which the next approximation to the solution is found as:

$$y^{k+1} = y^k + \alpha_k d^k$$

The relaxation factor $\alpha_j \epsilon (0,1]$ ensures global convergence; it results from a steplength algoirthm by properly reducing the test function.

$$R(y,\rho) = F(y) + \sum_{i \epsilon J_e} \rho_i |c_i(y)| + \sum_{i \epsilon J_i} \rho_i |\min[(c_i(y), 0]|$$

with suitably chosen parameters ρ. H is an approximation to the Hessian of the Lagrangian

$$\mathcal{L}(y,\lambda) = F(y) - \lambda^T c(y)$$

The algorithm belongs to the class of "variable metric methods for constrained optimization" and converges superlinearly.[10]

III. Application of the Method to a Simple Structure

The method proposed in Sec. II is applied to simple cantilevered gravity free beam, whose characteristic data are listed in the Appendix. The location of one actuator, one velocity sensor, and a scalar feedback gain were to be op-

Table 1 Optimal positions and feedback gains for an impulse surface load

Solution	Actuator, L	Sensor, L	Feedback	$\eta = \frac{W_c \cdot 100}{W_0}$	η_u, %
1	0.322	0.266	20.361	99.65	1.1
2	0.438	0.440	7.843	99.56	2.2
3	0.574	0.588	4.475	99.40	3.8
4	0.708	0.701	7.788	99.57	2.3
5	0.839	0.855	2.750	98.40	6.1
6	0.918	0.918	4.790	95.30	3.7
7	1.0	1.0	1.667	99.10	10.6

Table 2 Eigenvalues and damping factors for impulse surface load

Solution%	Eigenvalue locations (damping, %)				
1	−12.9;	−0.2 (100);	$-0.21\pm j7.33$ (2.9);	$-0.29\pm j2.53$ (11.3);	$-0.1\pm j0.316$ (32.4)
2	−3.25;	−0.14 (100);	$-0.6\ \pm j7.41$ (8.5);	$-0.19\pm j3.81$ (5.0;	$-0.25\pm j0.46$ (47.8)
3	−1.05;	−0.09 (100);	$-0.29\pm j7.73$ (3.7);	$-0.22\pm j3.88$ (5.8);	$-0.4\pm j0.98$ (38.9)
4	−3.92;	−0.03 (100);	$-0.32\pm j7.54$ (4.2);	$-0.76\pm j3.09$ (23.7);	$-0.08\pm j1.3$ (6.0)
5	−1.27;	−0.044 (100);	$-0.29\pm j7.76$ (3.7);	$-0.07\pm j3.97$ (1.9);	$-0.05\pm j1.37$ (3.5)
6	−3.87;	−0.019 (100);	$-0.02\pm j7.8$ (0.2);	$-0.0\pm j3.98$ (0.3);	$-0.08\pm j1.8$ (6.6)
7	−2.14;	−0.044 (100);	$-0.136\pm j7.77$ (1.7);	$-0.1\pm j_I 3.92$ (2.7);	$-0.2\pm j1.06$ (18.7)

timized according to Eq. (17), with x_a, x_s restricted to lie on the beam, that is,

$$0 \leq (x_{a_I}, x_{s_I}) \leq L \tag{25}$$

and

$$-100 \leq k_{II} \leq +100$$

For collocation (i.e., $x_{a_I} = x_{s_I}$), the limitation to one actuator and one sensor causes D_c to be positive definite for positive k_{II}. No gyroscopic term results in this case, and the system remains stable even for the residual modes, as $D_c \geq 0$ is together with $D_0 > 0$ a sufficient condition for stability.

As disturbance force to be investigated, an impulse surface load acting on the beam was chosen, which gave the following initial conditions of the generalized coordinate vector:

$$q(0)^T = [0\ 0\ 0\ 0]$$

$$\dot{q}(0)^T = [0.525\ \ 0.292\ \ 0.171\ \ 0.122]$$

with the total initial energy $W_0 = 0.203$.

Normalizing the dissipation energy with W_0, that is, optimizing $\eta = W_c(x_{aI}, x_{sI}, k_{II}) / W_0$, which in the following is referred to as overall efficiency, the solutions of Table 1 are determined, where the control efficiency η_u is determined by

$$\eta_u = W_c(x_{a_I}, x_{s_I}, k_{II}) \bigg/ \int_{t_0}^{\infty} f^T f \mathrm{d}t$$

The results of the optimization with a set of 25 initial values for x_{a_I}, x_{s_I}, and k_{II} are tabulated in Table 1.

For this low-order example with three variables and six constraints, on the average, 15 iterations with a total of 100 function calls were necessary. This took in the order of about 5-10 s per parameter set on a IBM 3081 mainframe system.

The solutions are mostly dislocated, thus resulting in gyroscopic feedback terms. Due to these gyroscopic terms more energy can be dissipated than for collocated design. Solution 1 is the global optimum, the others are local optima, all separated by a zero crossing of a neighboring mode shape (see Fig. 1).

The dissipated energy, respectively the efficiency η, is rather low for solution 6, because the positions are close to the zero crossings of modes 3 and 4. The best efficiency occurs for solution 1, which is the global optimum. Solutions 2 and 4, however, differ only slightly. Preference for either one of them or for solutions 3 and 7 depends on further characteristic data of the solutions, such as, for example, the control efficiency specified. Therefore the resulting eigenvalue locations and corresponding damping ratios are displayed in Table 2.

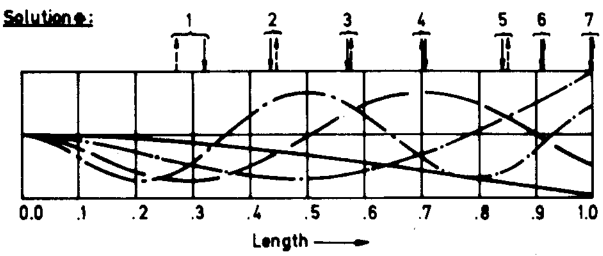

Fig. 1 Mode shapes and optimal →actuator ←--sensor positions.

A good damping behavior is achieved for solutions 1-3, where high damping exists for the lower modes and sufficient damping for the higher ones. Considering the T_{95} decay time, which specifies the decay time of an amplitude to 0.05 of its initial value, good damping would require that oscillations are damped out after at least three periods of the lowest frequency.

This leads to real parts of at least −0.0361 for real eigenvalues and a damping of at least 17% for complex eigenvalues for the lower modes. These requirements are fulfilled for solutions 1-3 and 7, which indeed exhibit a sufficient time behavior of the beam after impact of the impulse surface load as shown in Figs. 2-9. The four curves in each figure represent the motion of four distinguished points of the beam, namely, the points $l_1 = 1.0L$, $l_2 = 0.75L$, $l_3 = 0.5L$, and $l_4 = 0.25L$.

The largest deviation from the equilibrium normally occurs at l_1, the second largest at l_2, and so on (for example, see Figs. 3-5). However, in other figures the deviation of an interior point of the beam may be larger than the deviation of the free end of the beam.

For the final choice of the control to be realized a tradeoff between, for example, amplitude, decay time, control energy, and control amplitude has to be done which is beyond the

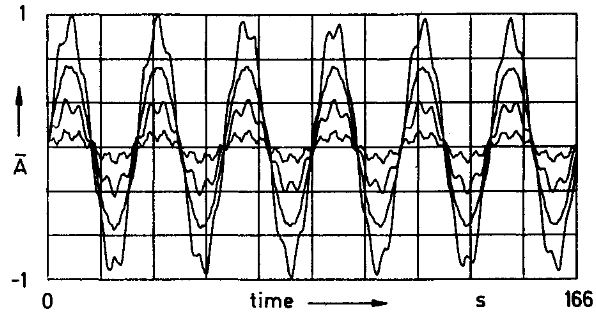

Fig. 2 Open loop behavior.

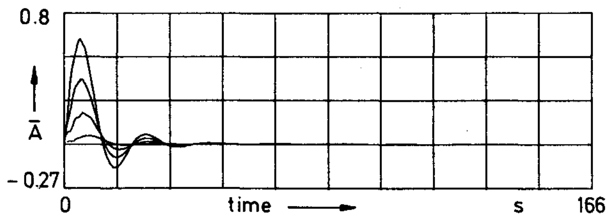

Fig. 3 Time behavior, solution 1.

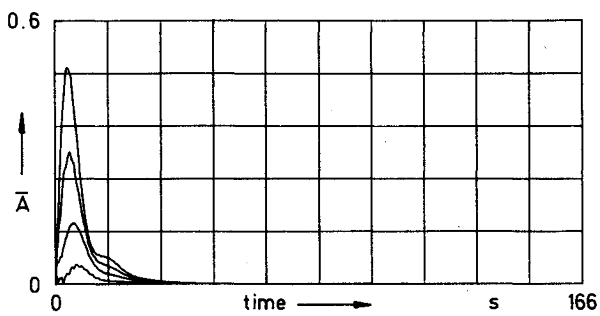

Fig. 4 Time behavior, solution 2.

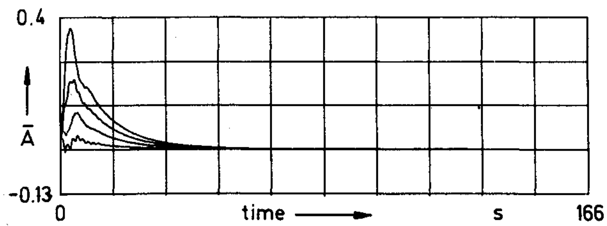

Fig. 5 Time behavior, solution 3.

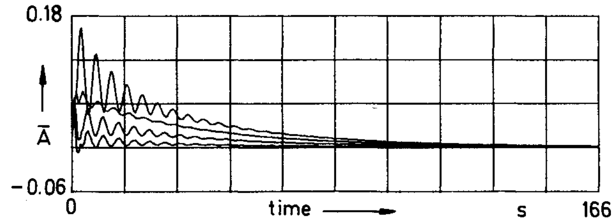

Fig. 6 Time behavior, solution 4.

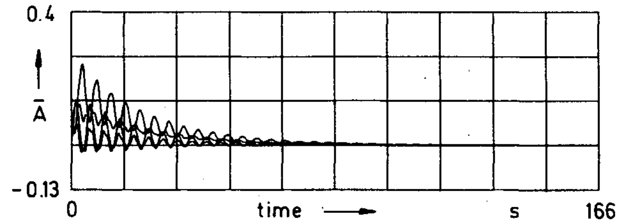

Fig. 7 Time behavior, solution 5.

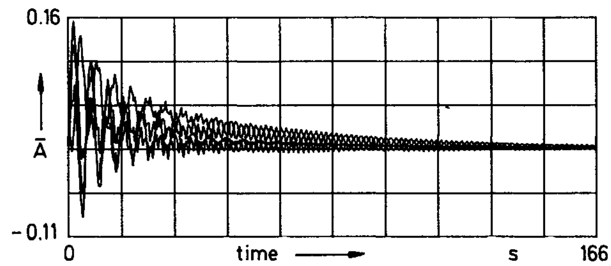

Fig. 8 Time behavior, solution 6.

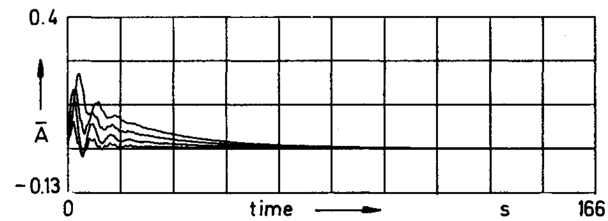

Fig. 9 Time behavior, solution 7.

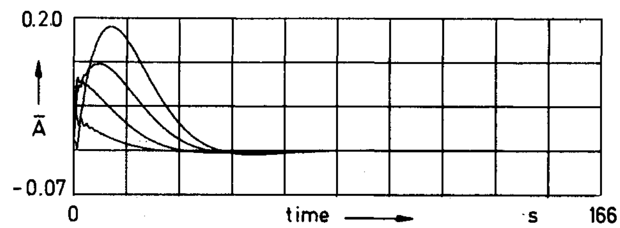

Fig. 10 Time behavior for two actuators and sensors.

scope of this paper. Practical constraints such as bounded control energy or numerically given mode shapes can be implemented easily. As approximation to bounded control energy, limitations of the feedback gains can be used. Numerically given mode shapes can be applied if table look-ups with intermediate interpolation yield values as good as analytically given mode shapes.

This low-order model with one actuator and one sensor is used as an introductory presentation of the method, which has been applied successfully to higher-order models (Darper model No. 1) with 6 actuators, 6, sensors, and 36 feedback gains.[13]

IV. Extensions of the Proposed Method to Treat Spillover Effects

The results of the last section indicate that the positions of actuators and sensors in general are dislocated. This may lead to instability as soon as truncated residual modes are implemented in the model and have a zero crossing between actuator and sensor positions. This instability does not occur for collocated actuator and sensor positions and positive definite feedback matrices.

In general, one intends to make a design for a low-order system with only very few modes because of accuracy and calculation time restrictions. But, at the same time, this design should guarantee stability for a higher-order system with a larger number of modes. Applying the above optimization method, we find a simple extension which allows us to utilize this desirable feature.

In addition to the optimization,

$$\max_{x_a,x_s K} W_c(x_a,x_s,K) \longrightarrow x_a^*,x_s^*,K^* \qquad (26)$$

with constraints

$$x_a \epsilon X_a, x_s \epsilon X_s, (K \epsilon K) \tag{27}$$

the additional constraint

$$D_c^* \geq 0 \tag{28}$$

is required.

Thereby

$$D_c^* = \tfrac{1}{2}\{B^*(x_a)KC^*(x_s) + [B^*(x_a)KC^*(x_s)]^T\} \tag{29}$$

represents the damping matrix due to the action of feedback for the higher-order model where $D_c^* \geq 0$ is sufficient for stability.

Thereby $B^*(x_a)$ and $C^*(x_s)$ represent the corresponding B and C matrices for a system with, for example, n^* number of modes, where $n^* > n$.

The design technique with $n^* = 5$ is applied to the same beam with two actuators and sensors and three free parameters of the K matrix. The initial damping was $\xi_i = 1\%$ for $i = 1,...,5$, whereby the first four modes ($n = 4$) were used in the optimization. Then locally the following collocated results were achieved:

Actuators: $x_a^T = (0.594 \quad 0.895)$

Sensors: $x_s^T = (0.594 \quad 0.895)$

Feedback: $K = \begin{bmatrix} 5.271, & -10.579 \\ 0 & 5.341 \end{bmatrix}$

The corresponding time behavior for an impulse surface load is displayed in Fig. 10.

Contrary to the above-obtained dislocated solutions, this one taking care of spillover effects yields collocated results. This seems to weaken the discussion on dislocated results as spillover has to be taken into account. However, the importance of collocation is based on the fact that together with a symmetric positive definite gain K, even the unmodeled modes remain stable.

Here optimization did not yield a symmetric feedback gain although collocation of actuators and sensors was obtained. Thus again, as in the single-input single-output case the feedback matrix K contained a skew symmetric term G_c of conservative forces. This gyroscopic term (induced here by the unsymmetry of K, while in the former chapter induced by dislocation) increased the amount of energy dissipated by controller action in comparison with a pure dissipative feedback $K \equiv D_c$.

V. Conclusions

In this paper a new method is presented which allows an integrated determination of actuator/sensor positions and feedback gains of flexible structures. This method is based on the maximization of dissipation energy due to control action. The optimality criterion is determined via an efficient solution of a Liapunov equation, and it is maximized with a recursive quadratic programming algorithm that allows to implement linear and nonlinear constraints. The application of this method to a simple flexible structure yields several in-general dislocated actuator and sensor locations, which are locally optimal. The number of local solutions depends on the number of zero crossings of the modeled mode shapes. An extension of the method to treat spillover effects is implemented as an additional constraint to the optimization criterion. The ongoing research covers multiple actuator/sensor locations and will also consider dynamic compensators in the feedback loop.

Appendix—Flexible Structure Specifications

Characteristic data of a cantilevered gravity free beam of length L:

mass/length $\bar{m} = 1.49$ kg/m

length $L = 3.81$ m

modulus of elasticity $E = 2.07 \cdot 10^5$ N/mm^2

moment of inertia $J = 6.35$ mm^4

stiffness $EJ = 1.31$ Nm^2

Eigenfrequency of the first four modes:

$\omega_1 = 0.227$ rad/s

$\omega_2 = 1.42$ rad/s

$\omega_3 = 3.99$ rad/s

$\omega_4 = 7.85$ rad/s

Damping factor $\xi_i = 0.1\%$ for $i = 1,...,4$.

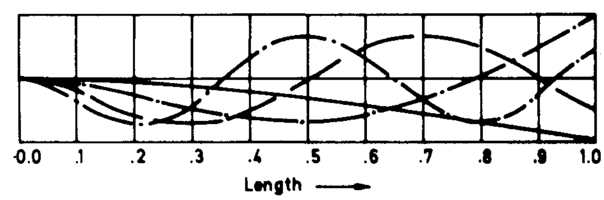

Fig. A1 Mode shapes of the first four modes: (#1_", #2 "– · –," #3 "– –,", #4 "– - –").

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